Framework for the Heart Rate Variability Analysis. Modeling the Pedalling Frequency: Effect of the Jitter and the Lack of Synchronisation

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RÉSUMÉ :

Dans ce rapport, on propose un modèle de génération d'un processus stochastique a partir du mouvement des deux jambes lors d'un exercice de pédalage. On montre qu'il est stationnaire et que le rapport d'amplitude du fondamental par rapport à la premiere harmonique est, en autre, fonction d'un retard systématique entre les deux jambes.

MOTS CLÉS :

processus stochastique, rythme cardiaque à l'effort, fréquence de pédalage

ABSTRACT:

In this report, we investigate the relation between the movement of the two legs during a pedaling exercise and the stochastic process that is observed. We show that it is stationary and that the ratio of the magnitude of the fundamental frequency and its first harmonic is at least a function of a constant delay between the two legs.

KEY WORDS :

stochastic process, heart rate variability during exercise, pedaling frequency

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During a pedaling exercise bout, the venous systems of the two legs induce a time varying feedback toward the heart. This variation corresponds to the alternating effort exhibited by each leg. The proposed model assumes a periodicity composed by a fundamental frequency $x_1(t)$ added to a harmonic component $x_2(t)$. Note that the more the effort is sinusoidal the less the magnitude of the harmonic component is. We assume that the right and left efforts are in opposite phase plus a random delay called jitter and a systematic bias. This produces the following mathematical model :

$$x(t) = x_1(t) + x_2(t) \tag{1}$$

with

$$x_1(t) = a_1 \cos(2\pi f_0(t + \tilde{d}(t)) + \varphi) + b_1 \cos(2\pi f_0 t + \varphi) \tag{2}$$

$$x_2(t) = a_2 \cos(2\pi 2 f_0(t + \tilde{d}(t)) + 2\varphi) + b_2 \cos(2\pi 2 f_0 t + 2\varphi) \tag{3}$$

The random delay between the legs is defined by :

$$\tilde{d}(t) = \bar{d} + d(t) + \frac{1}{2f_0} \tag{4}$$

where $d(t)$ is a centered random variable and φ a uniform random phase between 0 and 2π.

The calculation of $E[x(t)x(t+\tau)]$ needs to develop the sum :

$$\begin{aligned}
&E[x(t)x(t+\tau)] \\
&\quad = E[x_1(t)x_1(t+\tau)] + E[x_2(t)x_2(t+\tau)] + E[x_1(t)x_2(t+\tau)] + E[x_2(t)x_1(t+\tau)] \\
&\quad = E_1 + E_2 + E_3 + E_4
\end{aligned}$$

In the sequel, we are going to calculate individually the terms appearing in the previous expression.

For E_1, the product becomes :

$$a_1^2 \cos(2\pi f_0(t + \tilde{d}(t)) + \varphi) \cos(2\pi f_0(t + \tau + \tilde{d}(t+\tau)) + \varphi) \tag{5}$$

$$+a_1 b_1 \cos(2\pi f_0(t + \tilde{d}(t)) + \varphi) \cos(2\pi f_0(t+\tau) + \varphi) \tag{6}$$

$$+a_1 b_1 \cos(2\pi f_0 t + \varphi) \cos(2\pi f_0(t + \tau + \tilde{d}(t+\tau)) + \varphi) \tag{7}$$

$$+b_1^2 \cos(2\pi f_0 t + \varphi) \cos(2\pi f_0(t+\tau) + \varphi) \tag{8}$$

Using the result in (25), the expectation of the term (8) is :

$$E[(8)] = \frac{1}{2} b_1^2 \cos(2\pi f0\tau) \tag{9}$$

The term (6) is extended as :

$$\begin{aligned}
&(6) \rightarrow \\
&\quad a_1 b_1 \cos(2\pi f_0 \tilde{d}(t)) \cos(2\pi f_0 t + \varphi) \cos(2\pi f_0(t+\tau) + \varphi) \\
&\quad - a_1 b_1 \sin(2\pi f_0 \tilde{d}(t)) \sin(2\pi f_0 t + \varphi) \cos(2\pi f_0(t+\tau) + \varphi)
\end{aligned}$$

The random variables $\tilde{d}$ et φ being independent, using (25) and (27) we get :

$$E[(6)] = \frac{a_1 b_1}{2} E[\cos(2\pi f_0 \tilde{d}(t))] \cos(2\pi f_0 \tau) + \frac{a_1 b_1}{2} E[\sin(2\pi f_0 \tilde{d}(t))] \sin(2\pi f_0 \tau) \tag{10}$$

Using (4), we develop the terms $\cos(2\pi f_0 \tilde{d}(t))$ and $\sin(2\pi f_0 \tilde{d}(t))$, giving :

$$\cos(2\pi f_0 \tilde{d}(t)) = \cos(2\pi f_0 \bar{d}) \cos(2\pi f_0 d(t) + \pi) - \sin(2\pi f_0 \bar{d}) \sin(2\pi f_0 d(t) + \pi) \tag{11}$$
$$\sin(2\pi f_0 \tilde{d}(t)) = \sin(2\pi f_0 \bar{d}) \cos(2\pi f_0 d(t) + \pi) + \cos(2\pi f_0 \bar{d}) \sin(2\pi f_0 d(t) + \pi) \tag{12}$$

Assuming that the variations of $d(t)$ are small enough, we get the approximations :

$$\cos(2\pi f_0 d(t) + \pi) \approx -1 \; et \; \sin(2\pi f_0 d(t) + \pi) \approx -2\pi f_0 d(t) \tag{13}$$

The random variable $d(t)$ being zero mean, using the previous approximations the expression (10) becomes :

$$\begin{aligned} E[(6)] &= -\frac{a_1 b_1}{2} \cos(2\pi f_0 \bar{d}) \cos(2\pi f_0 \tau) - \frac{a_1 b_1}{2} \sin(2\pi f_0 \bar{d}) \sin(2\pi f_0 \tau) \\ &= -\frac{a_1 b_1}{2} c_1 \cos(2\pi f_0 \tau) - \frac{a_1 b_1}{2} s_1 \sin(2\pi f_0 \tau) \end{aligned} \tag{14}$$

The term (7) is developed as :

$$\begin{aligned} &(7) \rightarrow \\ &\quad a_1 b_1 \cos(2\pi f_0 \tilde{d}(t+\tau)) \cos(2\pi f_0 (t+\tau) + \varphi) \cos(2\pi f_0 t + \varphi) \\ &\quad - a_1 b_1 \sin(2\pi f_0 \tilde{d}(t+\tau)) \sin(2\pi f_0 (t+\tau) + \varphi) \cos(2\pi f_0 t + \varphi) \end{aligned}$$

Since $\tilde{d}$ and φ are independent, using (25) and (26) we get :

$$E[(7)] = \frac{a_1 b_1}{2} E[\cos(2\pi f_0 \tilde{d}(t+\tau))] \cos(2\pi f_0 \tau) - \frac{a_1 b_1}{2} E[\sin(2\pi f_0 \tilde{d}(t+\tau))] \sin(2\pi f_0 \tau) \tag{15}$$

By using the results from (11), (12) and (13), we finally get :

$$E[(7)] = -\frac{a_1 b_1}{2} c_1 \cos(2\pi f_0 \tau) + \frac{a_1 b_1}{2} s_1 \sin(2\pi f_0 \tau) \tag{16}$$

The term (5) is develop as :

$$\begin{aligned} &(5) \rightarrow \\ &\quad a_1^2 [\cos(2\pi f_0 t + \varphi) \cos(2\pi f_0 \tilde{d}(t)) \cos(2\pi f_0 (t+\tau) + \varphi) \cos(2\pi f_0 \tilde{d}(t+\tau)) \\ &\quad - \cos(2\pi f_0 t + \varphi) \cos(2\pi f_0 \tilde{d}(t)) \sin(2\pi f_0 (t+\tau) + \varphi) \sin(2\pi f_0 \tilde{d}(t+\tau)) \\ &\quad - \sin(2\pi f_0 t + \varphi) \sin(2\pi f_0 \tilde{d}(t)) \cos(2\pi f_0 (t+\tau) + \varphi) \cos(2\pi f_0 \tilde{d}(t+\tau)) \\ &\quad + \sin(2\pi f_0 t + \varphi) \sin(2\pi f_0 \tilde{d}(t)) \sin(2\pi f_0 (t+\tau) + \varphi) \sin(2\pi f_0 \tilde{d}(t+\tau))] \end{aligned} \tag{17}$$

By using (25), (26), (27), (28), (11), (12) the expectation of (5) becomes :

$$\begin{aligned} &E((5)) \rightarrow \\ &\quad \frac{a_1^2}{2} [\cos(2\pi f 0 \tau) E[(-c_1 + s_1 2\pi f_0 d(t))(-c_1 + s_1 2\pi f_0 d(t+\tau))] \\ &\quad - \sin(2\pi f_0 \tau) E[(-c_1 + s_1 2\pi f_0 d(t))(-s_1 - c_1 2\pi f_0 d(t+\tau))] \\ &\quad + \sin(2\pi f_0 \tau) E[(-s_1 - c_1 2\pi f_0 d(t))(-c_1 + s_1 2\pi f_0 d(t+\tau))] \\ &\quad + \cos(2\pi f_0 \tau) E[(-s_1 - c_1 2\pi f_0 d(t))(-s_1 - c_1 2\pi f_0 d(t+\tau))] \end{aligned}$$

That is simplified as :

$$E[(5)] = \frac{a_1^2}{2}(1 + (2\pi f_0)^2 R_{DD}(\tau)) \tag{18}$$

with $R_{DD}(\tau)$ the correlation function of the process $d(t)$.

The term E_1 being the sum of (9), (14), (16), (18), we get :

$$R_{x_1x_1}(\tau) = E_1 = \frac{(a_1 2\pi f_0)^2}{2}\cos(2\pi f_0\tau)R_{DD}(\tau) + \frac{1}{2}(a_1^2 - 2a_1b_1c_1 + b_1^2)\cos(2\pi f_0\tau) \tag{19}$$

The term E_2 is computed similarly but using the approximation :

$$\cos(2\pi 2 f_0 d(t) + 2\pi) \approx +1 \; et \; \sin(2\pi 2 f_0 d(t) + 2\pi) \approx 2\pi 2 f_0 d(t) \tag{20}$$

Then we get :

$$R_{x_2x_2}(\tau) = E_2 = \frac{(a_2 2\pi 2 f_0)^2}{2}\cos(2\pi 2 f_0\tau)R_{DD}(\tau) + \frac{1}{2}(a_2^2 + 2a_2b_2c_2 + b_2^2)\cos(2\pi 2 f_0\tau) \tag{21}$$

Taking into account the same hypothesis than for the previous calculation and the results from (25), (26), (27), (28) we show that $E_3 = 0$ and $E_4 = 0$. We finally obtain that the process $x(t)$ is stationary with a correlation function $R_{xx}(\tau)$ equal to :

$$\begin{aligned} R_{xx}(\tau) &= \frac{1}{2}(a_1^2 - 2a_1b_1c_1 + b_1^2)\cos(2\pi f_0\tau) + \frac{1}{2}(a_1 2\pi f_0)^2\cos(2\pi f_0\tau)R_{DD}(\tau) \\ &+ \frac{1}{2}(a_2^2 - 2a_2b_2c_2 + b_2^2)\cos(2\pi 2 f_0\tau) + \frac{1}{2}(a_2 2\pi 2 f_0)^2\cos(2\pi 2 f_0\tau)R_{DD}(\tau) \end{aligned} \tag{22}$$

We Assume that $d(t)$ is a low-pass filtered white random noise with a variance σ^2 whose filter frequency response is $H(f)$. Introducing a ratio α between the effort of the two legs we get the relations $b_1 = \alpha a_1$ and $b_2 = \alpha a_2$ $(0 < \alpha < 1)$. Finally the power spectral density $X(f)$ of $x(t)$ is given by :

$$\begin{aligned} X(f) &= \frac{a_1^2}{4}(1 - 2\alpha c_1 + \alpha^2)(\delta(f - f_0) + \delta(f + f_0)) + (a_1\sigma\pi f_0)^2(|H(f - f_0)|^2 + |H(f + f_0)|^2) \\ &+ \frac{a_2^2}{4}(1 + 2\alpha c_2 + \alpha^2)(\delta(f - 2f_0) + \delta(f + 2f_0)) + (a_2\sigma\pi 2 f_0)^2(|H(f - 2f_0)|^2 \\ &+ |H(f + 2f_0)|^2) \end{aligned} \tag{23}$$

with $c_1 = \cos(2\pi f_0\bar{d})$ et $c_2 = \cos(2\pi 2 f_0\bar{d})$

The analysis of this result can be simplified by neglecting in a first time the components produced by the jitter (i.e. $\sigma = 0$). So, the magnitude ratio R of the functions δ at the fundamental frequency and its harmonic is given by :

$$R = \beta\sqrt{\frac{1 - 2\alpha c_1 + \alpha^2}{1 + 2\alpha c_2 + \alpha^2}} \tag{24}$$

where β is the ratio of the magnitude a_1 and a_2.

Defining $d_r = \frac{100}{T_0}\bar{d}$, the permanent delay expressed as a percentage of the pedaling period T_0, we get a set of curves function of d_r and α. The terms β is simply a scaling factor to be applied the the values plotted in fig. (1).

We can deduced from the curves that the fundamental component can be higher than those from its harmonic when the pedaling movement is close to a pure sinusoid, i.e. increasing β, and that a permanent delay is present as far as the exercise lasts, i.e. increasing d_r. Added to this fact, when the force exerted by the legs becomes unbalanced the α value decreases causing a reinforcement of the magnitudes difference. Additionally, when $a_1 > 2a_2$ this ratio is increased as the jitter magnitude is higher, i.e. increasing σ in (23).

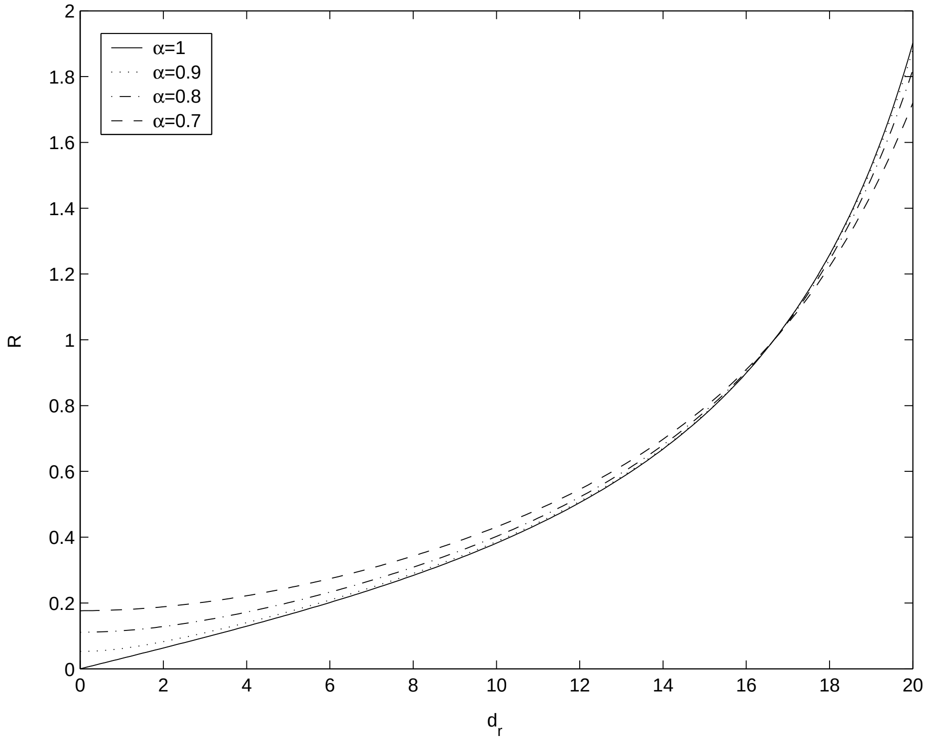

FIG. 1 – Curves R obtained for several values of α and d_r

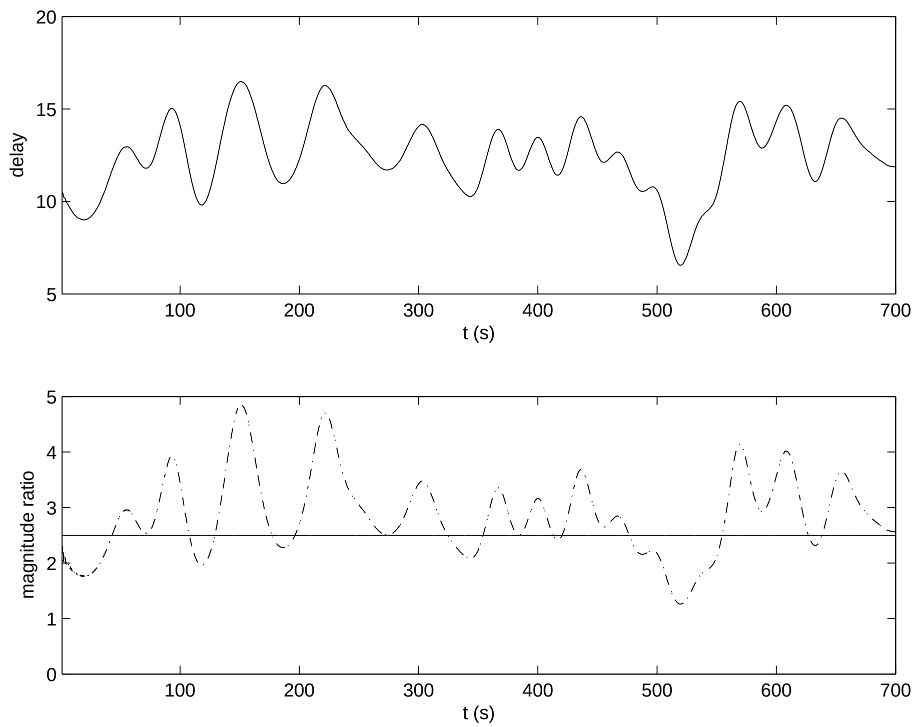

FIG. 2 – Time variation of the delay function of time (top). Approximated ratio R (solid line) and ratio of the magnitude (dash-dotted line) of the fundamental and harmonic components (2) (bottom)

To illustrate this result that is an average one, we give in fig. (2 top trace), an example of the function $\bar{d}+d(t)$ in percent of the pedaling period, with $d_r = 10\%$, $\alpha = 0.8$ and $a_1/a_2 = 5$. In the bottom trace, we compare the ratio of the magnitude of the fundamental frequency with the harmonic (2) and the value of R where the random jitter is not taken into account. It is clear that under such realistic conditions the component at the fundamental frequency is higher than its harmonic.

1 ANNEXE

Calculation of the correlation and cross-correlation functions used in the modelling.

In the sequel, we will use the property that φ is a uniform random variable defined on the interval $[0, \pi]$ and (n, m) a couple of integers > 0.

$$
\begin{aligned}
&E[\cos(n\omega t + n\varphi)\cos(m\omega(t+\tau) + m\varphi)] \\
&\quad = \int \cos(n\omega t + n\varphi)\cos(m\omega t + m\tau) + m\varphi) p_\phi(\varphi) d\varphi \\
&\quad = \frac{1}{4\pi}(\int_0^{2\pi} \cos(\omega t(n-m) - m\omega\tau + \varphi(n-m)) d\varphi + \int_0^{2\pi} \cos(\omega t(n+m) + m\omega\tau + \varphi(n+m)) d\varphi)
\end{aligned}
$$

The second term being equal to zero for any values of n and m, we finally get :

$$
R1: \; E[\cos(n\omega t + n\varphi)\cos(m\omega(t+\tau) + m\varphi)] = \begin{cases} 0 \; pour \; n \neq m \\ \frac{1}{2}\cos(n\omega\tau) \; pour \; n = m \end{cases} \tag{25}
$$

$$
\begin{aligned}
&E[\cos(n\omega t + n\varphi)\sin(m\omega(t+\tau) + m\varphi)] \\
&\quad = \int \cos(n\omega t + n\varphi)\sin(m\omega t + m\tau) + m\varphi) p_\phi(\varphi) d\varphi \\
&\quad = \frac{1}{4\pi}(\int_0^{2\pi} \sin(\omega t(m-n) + m\omega\tau + \varphi(m-n)) d\varphi + \int_0^{2\pi} \sin(\omega t(n+m) + m\omega\tau + \varphi(n+m)) d\varphi)
\end{aligned}
$$

The second term being equal to zero for any values of n and m, we finally get :

$$
R2: \; E[\cos(n\omega t + n\varphi)\sin(m\omega(t+\tau) + m\varphi)] = \begin{cases} 0 \; pour \; n \neq m \\ \frac{1}{2}\sin(n\omega\tau) \; pour \; n = m \end{cases} \tag{26}
$$

$$
\begin{aligned}
&E[\sin(n\omega t + n\varphi)\cos(m\omega(t+\tau) + m\varphi)] \\
&\quad = \int \sin(n\omega t + n\varphi)\cos(m\omega t + m\tau) + m\varphi) p_\phi(\varphi) d\varphi \\
&\quad = \frac{1}{4\pi}(\int_0^{2\pi} \sin(\omega t(n-m) - m\omega\tau + \varphi(n-m)) d\varphi + \int_0^{2\pi} \sin(\omega t(n+m) + m\omega\tau + \varphi(n+m)) d\varphi)
\end{aligned}
$$

The second term being equal to zero for any values of n and m, we finally get :

$$
R3: \; E[\sin(n\omega t + n\varphi)\cos(m\omega(t+\tau) + m\varphi)] = \begin{cases} 0 \; pour \; n \neq m \\ -\frac{1}{2}\sin(n\omega\tau) \; pour \; n = m \end{cases} \tag{27}
$$

$$
\begin{aligned}
&E[\sin(n\omega t + n\varphi)\sin(m\omega(t+\tau) + m\varphi)] \\
&\quad = \int \sin(n\omega t + n\varphi)\sin(m\omega t + m\tau) + m\varphi) p_\phi(\varphi) d\varphi \\
&\quad = \frac{1}{4\pi}(\int_0^{2\pi} \cos(\omega t(n-m) - m\omega\tau + \varphi(n-m)) d\varphi - \int_0^{2\pi} \cos(\omega t(n+m) + m\omega\tau + \varphi(n+m)) d\varphi)
\end{aligned}
$$

The second term being equal to zero for any values of n and m, we finally get :

$$R4: \ E[\sin(n\omega t + n\varphi)\sin(m\omega(t+\tau) + m\varphi)] = \left\{ \begin{array}{l} 0 \ pour \ n \neq m \\ \frac{1}{2}\cos(n\omega\tau) \ pour \ n = m \end{array} \right. \tag{28}$$